

(0,2) string compactifications

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Using the simple current method we study a class of $(0, 2)$ SCFTs which we conjecture to be equivalent to $(0, 2)$ σ models constructed in the framework of gauged linear sigma models.

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1. Introduction

It is well-known [1] that the classical string solutions with $N = 1$ spacetime supersymmetry require the internal CFTs to have $(0, 2)$ worldsheet supersymmetry. $(0, 2)$ models lead to more realistic gauge groups like $SO(10)$ or $SU(5)$ [2], and are therefore phenomenologically more interesting than the $(2, 2)$ case. To understand the space of ground states of classical string theory there is little reason to restrict oneself to the study of $(2, 2)$ models. But the assertion in [3] that the generic $(0, 2)$ σ model is destabilized by worldsheet instantons has slowed down further development in this direction. The revival of $(0, 2)$ σ models has been initiated mainly by the work of Distler and Greene [4,5], who gave a criterion for evading destabilization of the vacuum. Its verification is, however, not trivial. More recently Distler and Kachru [6] constructed a large class of $(0, 2)$ σ models that uses Witten's gauged linear sigma model approach [7]. The resulting class of models is now believed to define appropriate SCFTs at the infrared fixed point [8].

On the side of exact conformal models the situation seems to be more transparent. Using simple current techniques Schellekens and Yankielowicz [9] produced a telephone book of $(1, 2)$ models from tensor products of minimal models [10]. The main problem in this context is the arbitrariness in the selection of a manageable subset of the models that are accessible with the known construc-

tions [11,9]. This effort culminated in Schellekens' theorem on the conditions for the possibility of avoiding fractional electric charges [12,13].

An interesting question is, of course, to what extent one can identify the models constructed by these two approaches. Searching for such a class of models we will try to generalize the proposal of [14,15], which was originally made on the basis of a stochastic computer search for matching particle spectra. We will extend this to a series of identifications based on an analysis of the anomaly cancellation conditions and on the assumption that the \mathbb{Z}_4 breaking mechanism (see below) does not care about the rest of the conformal field theory and only acts on a Fermat factor of a non-degenerate Landau–Ginzburg superpotential. This provides us with 3219 $SO(10)$ models, based on the list of 7555 weights for transverse hypersurfaces in weighted projective spaces [16,17], and many more if we include orbifolding or hybrid constructions.

After a quick review of $(0, 2)$ compactification in the geometric context in section 2 we start with an ansatz to solve the anomaly cancellation equation and then briefly discuss some issues concerning the toric resolution of singularities [18–20]. In section 3 we discuss some aspects of exact constructions in the framework of simple currents and apply these to the class of models suggested by the σ model connection. We conclude with some comments about open problems and directions for future work.

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2. (0, 2) σ models and anomaly cancellation conditions

The geometric data that define a (0, 2) compactification [21, 2] consist of a Kähler manifold M with Kähler form J and some holomorphic vector bundle V of rank r . The right moving fermions on the worldsheet couple to the pull back of the tangent bundle TM along the sigma model map and the left moving ones couple to the pull back of V . The very existence of these spinors requires that the topological obstruction to their existence vanishes, i.e. $c_1(V) = c_1(M) = 0 \pmod{2}$ ³. Anomaly cancellation imposes a further condition on these data, namely $ch_2(M) = ch_2(V)$, where $ch_2 = \frac{1}{2}c_1^2 - c_2$ is the second Chern character. Vanishing of the lowest order beta functions requires that the Kähler metric is Ricci-flat, i.e. M is a Calabi-Yau manifold, and the connection on V has to satisfy the conditions $F_{ij} = F_{\bar{i}\bar{j}} = 0$ (i.e. it is holomorphic) and $g^{i\bar{j}}F_{i\bar{j}} = 0$. Due to the results of Donaldson and Uhlenbeck and Yau [2, 22] we have the following information about the set of solutions of $g^{i\bar{j}}F_{i\bar{j}} = 0$: If the integrability condition $\int_M J^{n-1} \wedge c_1(V) = 0$ is satisfied then the existence and uniqueness of the solution is equivalent to the stability of V with respect to J on M . The stability of V means that for every ‘coherent subsheaf’ \mathcal{F} of V with $0 < \text{rank}(\mathcal{F}) < \text{rank}(V)$ one has $\mu(\mathcal{F}) < \mu(V)$, where μ , called normalized first Chern number or slope, is defined by $\mu(\mathcal{F}) := \frac{1}{\text{rank}(\mathcal{F})} \int_M J^{n-1} \wedge c_1(\mathcal{F})$.

2.1. The gauged linear sigma models

It is believed that a ‘generic’ σ model of the type discussed above flows to a (0, 2) string vacuum in the infrared limit [2, 5]. As pointed out in [5], such models evade destabilization by world sheet instantons [3] if the splitting type of V on all instantons in the Calabi-Yau space M is nontrivial. However, it is technically difficult to check this condition. A real breakthrough has been achieved by Witten’s gauged linear sigma model approach [7]. This approach made it possible to construct a large class of (0, 2) string vacua that is field theoretically more manageable [6]. The start-

ing point is a (0, 2) supersymmetric $U(1)$ gauge theory that leads in the Calabi-Yau phase to a (0, 2) σ model described by the following geometric data:

$$0 \longrightarrow V \longrightarrow \bigoplus_{i=1}^{r+1} \mathcal{O}(n_i) \xrightarrow{F} \mathcal{O}(m) \longrightarrow 0 \quad (1)$$

is an exact sequence defining a stable bundle V of rank r on a complete intersection Calabi-Yau variety M in a weighted projective space $\mathbb{P}_{w_1, \dots, w_{N+1}}$. In the following we will abbreviate the vector bundle data by $V_{n_1, \dots, n_{r+1}}[m]$, where n_i are positive integers and F_i are homogeneous polynomials of degrees $m - n_i$ not vanishing simultaneously on M . The choices $r = 4$ and $r = 5$ correspond to unbroken gauge groups $SO(10)$ and $SU(5)$, respectively. As pointed out above these geometric data are subject to constraints which in this case lead to the following equations: Let M be the complete intersection of hypersurfaces of degrees d_j , $j = 1, \dots, N - 3$. Then the Calabi-Yau condition for M , i.e. $c_1(M) = 0$, leads to:

$$\sum_{j=1}^{N-3} d_j - \sum_{i=1}^{N+1} w_i = 0. \quad (2)$$

Taking the defining sequence of V into account, the condition $c_1(V) = 0$ means that

$$m - \sum_{i=1}^{r+1} n_i = 0. \quad (3)$$

The last condition, which comes from the cancellation of gauge anomalies, results in the quadratic diophantine equation

$$m^2 - \sum_{i=1}^{r+1} n_i^2 = \sum_{j=1}^{N-3} d_j^2 - \sum_{i=1}^{N+1} w_i^2. \quad (4)$$

For a Calabi-Yau hypersurface of degree d the choice $m = d$ and $n_i = w_i$ solves the above equations. In the special case of $F_i = \partial_i W$, where W is the transversal polynomial that defines M in $\mathbb{P}_{w_1, \dots, w_5}$, we have a (2, 2) model with gauge group E_6 . For a generic choice of F_i the bundle V will be a stable deformation of the extension of \mathcal{T}_M by \mathcal{O} . These models are therefore (0, 2) deformations of (2, 2) models (cf. [6] for more details).

³For a complex vector bundle the mod 2 reduction of c_1 is the second Stiefel–Whitney class.

2.2. A series of solutions

Our aim is to find a generalization of the map from a (0,2) SCFT to a set of vector bundle data that has been found in [14]. In that paper the weights n_i of a transverse CY hypersurface in weighted \mathbb{P}^4 correspond to the quintic (i.e. $n_i = 1$ and $m = \sum n_i = 5$) and it is these numbers that enter the definition of the vector bundle, whereas the base manifold gets modified and turns out to be the vanishing locus of two degree 4 polynomials in $\mathbb{P}_{1,1,1,1,2,2}^5$. At first glance it may be surprising that it is the base manifold and not the vector bundle data that changes. This is, however, in agreement with the fact that the discrete gauge symmetry that survives the breaking of the $U(1)$ in the gauged linear σ model is a \mathbb{Z}_m with m defined in eq. (1); this symmetry should be identified with the \mathbb{Z}_m quantum symmetry [23] that comes with the GSO projection on the CFT side.

As we will see in our discussion of the simple current construction of the (0,2) model, the breaking of the left-moving SUSY is accomplished by a twist that acts only on a Fermat factor of the tensor product constituting the internal SCFT and on an $SO(2)$ factor of the current algebra. This twist leads to a \mathbb{Z}_2 projection in the NS sector, which explains the doubling of the last weight and the requirement of an additional generator for the cohomology to describe the contributions from the twisted sectors. This additional generator is obtained by increasing the dimension of the ambient space and hence the codimension of the base manifold.

We are thus lead to the ansatz $V_{n_1, \dots, n_5}[m] \rightarrow \mathbb{P}_{n_1, \dots, n_4, w_5, w_6}[d_1, d_2]$ for the base manifold and vector bundle data, which we expect to work whenever there exists a transverse polynomial of degree m in four fields ϕ_i with weights n_1, \dots, n_4 and with $K = m/n_5$ being an odd integer (i.e. the internal CFT should be a tensor product with a Fermat factor corresponding to a minimal model at odd level). It is a non-trivial check on the existence of the desired series of maps that there are *positive integer* solutions to the anomaly equations (2–4). Demanding that the quadratic equation (4) factorizes we indeed find a unique (up to permutation of weights) general solution with $w_5 = 2n_5$, $2w_6 = d_1 = m - n_5$ and

$d_2 = (m + 3n_5)/2$. It is encouraging that the expected doubling of the Fermat weight comes about automatically. Searching through the list of 7555 transverse weights [16,17] we find 3219 LG potentials with 4 fields and central charge $c = 6(1 + 1/K)$ with $K \in 2\mathbb{Z} + 1$ being an odd integer. 220 of these have the property that all weights n_i divide the degree m , so that they correspond to tensor products of minimal models.

In the Calabi–Yau phase of our σ model we may be confronted with singularities in both, the base manifold and the vector bundle [20]. It is natural to try a toric approach to the resolution of these singularities. A canonical Calabi-Yau variety can be realized as a complete intersection in a Gorenstein Fano toric variety in the following way [18,24,19]: Using the correspondence of reflexive polytopes and Gorenstein Fano toric varieties we begin with a reflexive polytope Δ in a lattice of rank n and construct its corresponding toric variety $X = X_\Delta$ [24]. Next we consider a nef partition of the anticanonical divisor $-K_X = \sum_{\alpha=1}^N D_\alpha$ of X , which is a partition of $-K_X$ into a sum of nef Cartier divisors $D_j = \sum_{\alpha \in I_j} D_\alpha$ with $\bigcup_{j=1}^r I_j$ being a partition of $\{1, \dots, N\}$. If Δ_j are the polytopes associated to D_j it follows that $\Delta = \Delta_1 + \dots + \Delta_r$ is their Minkowski sum. Now let Y_j be a generic section of $\mathcal{O}_X(D_j)$. Then the complete intersection $\bigcap_{j=1}^r Y_j$ will be a canonical Calabi-Yau variety of codimension r in X .

To obtain a toric resolution of the singularities in the base manifold it is clear that we should choose Δ_i to be (subpolytopes of) the Newton polytopes corresponding to degree d_i monomials in ϕ_1, \dots, ϕ_6 with $i = 1, 2$. Obviously $\Delta_1 + \Delta_2$ is then a subpolytope of the Newton polytope corresponding to the weights $n_1, \dots, n_4, w_5, w_6$, but it is not clear under what conditions such a Δ will be reflexive and it may be necessary to study the situation on a case by case basis. There is no space to go into further details at this point [25].

3. Simple currents, modular invariants and orbifolds

In this section we recall some elements of the classification of simple current modular invari-

ants, which will allow us to give an orbifold interpretation to the construction of [14] and to derive explicit formulas for the spectra in terms of the *extended Poincaré polynomial* [26,27] (which is related to the elliptic genus).

A simple current J is a primary field that has unique fusion products with all other primary fields, and thus decomposes the field content of a CFT into orbits of finite length [28]. The order N of J is the length of the orbit of the identity, i.e. $J^N = \mathbf{1}$. Associativity and commutativity of the fusion product imply that the simple currents form an abelian group, which is called the center of the CFT. The important fact is that we can define a *monodromy charge*

$$Q_J(\phi) \equiv h_J + h_\phi - h_{J \times \phi} \pmod{1} \quad (5)$$

that is conserved modulo 1 in operator products. Since $e^{2\pi i Q_J}$ is conserved in OPEs, a simple current is thus always associated to a discrete \mathbb{Z}_N symmetry of the CFT and the center has a representation in terms of phase symmetries. The definition of Q_J implies

$$Q_{J \times K}(\phi) \equiv Q_J(\phi) + Q_K(\phi), \quad (6)$$

so that $Q_{J^n}(\phi) \equiv nQ_J(\phi)$ and Q_J is quantized in units of $1/N_J$.

For some fixed subgroup \mathcal{G} of the center that is generated by independent simple currents J_i of order N_i we use the notation $[\alpha] = \prod J_i^{\alpha_i}$ and $Q_i = Q_{J_i}$, where α_i are integers that are defined modulo N_i . Then we can parametrize the charges and conformal weights of all simple currents in \mathcal{G} in terms of a matrix R_{ij} [29],

$$R_{ij} = r_{ij}/N_i \equiv Q_i(J_j) \equiv Q_j(J_i), \quad (7)$$

$$h_{[\alpha]} \equiv \frac{1}{2} \sum_i r_{ii} \alpha^i - \frac{1}{2} \sum_{ij} \alpha^i R_{ij} \alpha^j, \quad (8)$$

with $r_{ij} \in \mathbb{Z}$. If N_i is odd we can choose r_{ii} to be even. Then all diagonal elements R_{ii} are defined modulo 2 for both, even and odd N_i . Using the definitions of Q and R we obtain

$$h_{[\alpha]\phi} \equiv h_\phi + h_{[\alpha]} - \alpha^i Q_i(\phi), \quad (9)$$

$$Q_i([\alpha]\phi) \equiv Q_i(\phi) + R_{ij} \alpha^j. \quad (10)$$

If N_i is even then J_i can occur in a modular invariant only if $r_{ii} \in 2\mathbb{Z}$ (since T -invariance requires $h_a - h_{J_i a} \in \mathbb{Z}$)⁴. In that case only a subgroup of the center that consists of simple currents J_i whose parameters r_{ii} all are even can contribute to a modular invariant. For such a group it can be shown that the matrix

$$M_{\phi, [\alpha]\phi} = \mu(\phi) \prod_i \delta_{\mathbb{Z}}(Q_i(\phi) + X_{ij} \alpha^j) \quad (11)$$

commutes with the generators S and T of modular transformations if X is properly quantized and $X + X^T \equiv R$ (for the diagonal $2X_{ii} \equiv R_{ii}$ must hold modulo 2) [30]. In this formula $\delta_{\mathbb{Z}}(r)$ is 1 if $r \in \mathbb{Z}$ and 0 otherwise, and $\mu(\phi)$ is the multiplicity of the primary field ϕ on its orbit. Under certain assumptions (11) can be shown to be the most general modular invariant that only relates primary fields on orbits of the center [30,29].

It is easy to see that the left and right chiral algebra extensions are given by the kernels $\mathcal{A}_R \cong \text{Ker}_{\mathbb{Z}} X$ and $\mathcal{A}_L \cong \text{Ker}_{\mathbb{Z}} X^T$, respectively, where $\text{Ker}_{\mathbb{Z}} M$ denotes the set of integer vectors α whose product with the matrix M is integer. (\mathcal{A}_R and \mathcal{A}_L have the same dimension, but need not be isomorphic [30].)

3.1. Symmetry breaking and GSO

An important application of the chiral algebra extension mechanism that we just discussed is the Ramond/NS sector alignment in the superconformal tensor product, which can be understood as a simple current modular invariant because the supercurrent J_v of any superconformal field theory is a simple current with $h = 3/2$ and order 2. Its monodromy charge is 0 in the NS sector and $1/2$ in the Ramond sector. In the conventional tensor product of two SCFTs we thus have the current $J_v^{(12)} = J_v^{(1)} \times J_v^{(2)}$, which has integer spin and can extend the chiral algebra. The monodromy parameter R_{vv} is indeed 0 modulo 2, so that the modular invariant (11) keeps only states with integer charge w.r.t. $J_v^{(12)}$, i.e. both factors of a field in the tensor product must come from the same sector.

⁴ This is related to the distinction between odd and even orders in the orbifold level matching conditions.

The GSO projection can be implemented in a similar way: In any $N = 2$ SCFT the Ramond ground state J_s with maximal charge is a simple current (which implements the spectral flow; for details see [27]). Therefore $J_{GSO} = J_s \times s$, with s being the spinor of $SO(10)$, is a simple current with spin 1 that extends the gauge group to E_6 and leads, after the string map, to space-time SUSY.

For a tensor product of n SCFTs we have to put all bilinears in the respective supercurrents into the chiral algebras [26,31]. In this case we have more freedom since the modular invariant is given by a $(n-1) \times (n-1)$ matrix X for which only the symmetric part has to vanish. If we choose $E_{ij} = \frac{1}{2}(X_{ij} - X_{ji}) = 0$ then $X \equiv 0$ and we have the maximal extension of the chiral algebras and complete alignment of R and NS sectors for both chiral halves. If $E \neq 0$ then some bilinears are projected out and SUSY is broken on both sides. We can construct models where the alignment is kept on the right-moving side if we have a larger center with additional simple currents of even order. This is one possible mechanism to construct $(0,2)$ models with gauge group $SO(10)$, which in general is not extended to E_6 by the GSO projection if the left-moving SUSY is broken.

The mechanism for the construction of $(0,2)$ models that is most natural from the point of view of σ models [21,2] is closely related to what we just discussed, but also involves simple currents of the gauge group. Since our aim is to construct a heterotic string via the string map applied to the right-moving sector we need to keep the $SO(10) \times E_8$ on that side and thus have to use some asymmetric construction.

The simple current way to break a symmetry is to start from building blocks that belong to a smaller chiral algebra and to view the CFT with broken symmetry as a modular invariant with ‘non-maximal’ extension of the chiral algebra. We therefore start with a 4-dimensional bosonic string with an internal $N = 2$, $c = 9$ SCFT and a current algebra $SO(2) \times SO(8) \times E_8$, which can be extended to $SO(10) \times E_8$ by putting the product $v \times V$ of the vector currents of $SO(2)$ and $SO(8)$ into the chiral algebra, as we did for the Ramond sector alignment above (we denote

the $SO(2)$ and $SO(8)$ representations by small and capital letters, respectively).

If we avoid that extension on the right-moving side we can still construct a modular invariant heterotic string with the string map, but the GSO projection on the left-moving side will in general extend $U(1) \times SO(8)$ only to $SO(10)$, where the $U(1)$ factor is a combination of the $SO(2)$ and the $U(1)$ of the $N = 2$ algebra. Non-abelian gauge groups of smaller rank are obtained by splitting $SO(10)$ into several $SO(2)$ factors.

3.2. $SO(10)$ models

Returning to the $SO(10)$ model based on the quintic that was conjectured to be equivalent to a Distler–Kachru model in [14] we first need to bring the modular invariant that enters the construction into the normal form (11). In addition to the alignment currents and the GSO projection there is only one more simple current involved, namely the order 4 current $J_b = \mathbf{1} \times \phi_K^{01} \times s \times \mathbf{1}$, i.e. the product of the identities in the CFT with $c = 6(1 + 1/K)$ and in the $SO(10)$ factor with the primary field labeled by $\{l = 0, q = K, s = 1\}$ of the minimal model at level $k = K - 2$ and the spinor of $SO(2)$. A simple calculation shows that all monodromies among these currents vanish for odd k except for $Q_b(J_A) = Q_A(J_b) = 1/2$, where $J_A = \mathbf{1} \times \mathbf{1} \times v \times V$ is the alignment current for $SO(2)$ and $SO(8)$. J_b thus indeed prohibits a simultaneous extension to $SO(10)$ on the left and on the right. Note that $J_b^2 = \mathbf{1} \times J_v^F \times v \times \mathbf{1}$ is the alignment current for the minimal model and the $SO(2)$, so that J_b can be regarded as a square root of an alignment current.

Since the conformal weight $h_b = (K - 1)/2$ of J_b depends on the level it is convenient to replace it by its product J_B with the alignment current $J_b^2 J_A$. This product has integer spin. In the resulting basis

$$J_{GSO} = J_s^C \times J_s^F \times s \times S \quad (12)$$

$$J_A = \mathbf{1} \times \mathbf{1} \times v \times V \quad (13)$$

$$J_B = \mathbf{1} \times (J_s^F)^K \times s \times (V)^{\frac{K-1}{2}} \quad (14)$$

$$J_C = J_v^C \times \mathbf{1} \times \mathbf{1} \times V \quad (15)$$

we find a monodromy matrix R whose only non-vanishing entries are $R_{AB} = R_{BA} = 1/2$.

We now need to choose discrete torsions in such a way that J_{GSO} and the alignment currents J_A , J_B^2 and J_C all are in the right-chiral algebra, i.e. all respective columns of X should vanish. This uniquely fixes the anti-symmetric part of X with $X_{AB} = 1/2$ being the only non-vanishing entry. Without further calculation we can thus conclude that this must be the invariant that enters the model of [14] in the special case of the quintic. It is straightforward to derive a formula for the numbers of $SO(10)$ representations for general $(0,2)$ compactifications of this type in terms of the extended Poincaré polynomial of the internal CFT [25]. What remains to be done is to check the matching of these data with the conjectured σ -model twin and to provide further evidence that the $(0,2)$ CFT indeed is in the same moduli space.

4. Conclusion

We used an ansatz for the $(0,2)$ σ model data that is inspired by the orbifold interpretation of a ‘prototype model’ [14] to find a large class of solutions to the anomaly cancellation conditions. The resulting analogies between the coordinate ring structure on the geometrical side and the orbifold selection rules on the CFT side indeed support the conjecture that this provides a series of identifications among apparently different constructions of $(0,2)$ compactifications.

Only a minimal model factor of the internal $c = 9$ SCFT takes part in the symmetry breaking mechanism, so that we can use arbitrary $N = 2$ SCFTs with $c = 6(1 + 1/K)$ to construct ‘hybrid models’, whose numbers of non-singlet massless representations can be computed in terms of their extended Poincaré polynomials [27].

On the sigma model side the problem of resolving singularities provides a number of new mathematical challenges [20,25]; the Landau–Ginzburg side of the triality has been studied for a closely related class of models in the recent paper [32]. Models with other gauge groups like $SU(5)$ can be constructed following the same lines.

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REFERENCES

1. T.Banks, L.J.Dixon, D.Friedan, E.Martinec, Nucl. Phys. **B299** (1988) 613
2. E.Witten, Nucl. Phys. **B268** (1986) 79
3. M.Dine, N.Seiberg, X.G.Wen, E.Witten, Nucl. Phys. **B278** (1986) 769, **B289** (1987) 319
4. J.Distler, Phys. Lett. **B188B** (1987) 431
5. J.Distler, B.Greene, Nucl. Phys. **B304** (1988) 1
6. J.Distler, S.Kachru, Nucl. Phys. **B413** (1994) 213
7. E.Witten, Nucl. Phys. **B403** (1993) 159
8. E.Silverstein, E.Witten, Nucl. Phys. **B444** (1995) 161
9. A.N.Schellekens and S.Yankielowicz, Nucl. Phys. **B330** (1990) 103
10. A.N.Schellekens and S.Yankielowicz, *Tables Supplements* to ref. [9] CERN-TH.5440S/89 and CERN-TH.5440T/89 preprints (unpublished)
11. A.Font, L.E.Ibáñez, M.Mondragon, F.Quevedo, G.G.Ross, Phys. Lett. **B227** (1989) 34
A.Font, L.E.Ibáñez, F.Quevedo, A.Sierra, Nucl. Phys. **B337** (1990) 119
12. A.N.Schellekens, Phys. Lett. **B237** (1990) 363
13. J.D.Lykken, hep-th/9607144
14. R.Blumenhagen, A.Wiřkirchen, Nucl. Phys. **B454** (1995) 561; Nucl. Phys. **B475** (1996) 225
15. R.Blumenhagen, R.Schimmrigk, A.Wiřkirchen, Nucl. Phys. **B461** (1996) 460
16. M.Kreuzer, H.Skarke, Nucl. Phys. **B388** (1992) 113
17. A.Klemm, R.Schimmrigk, Nucl. Phys. **B411** (1994) 559
18. L.A.Borisov, alg-geom/9310001
19. V.V.Batyrev, L.A.Borisov, alg-geom/9402002; alg-geom/9412017; alg-geom/9509009
20. J.Distler, B.R.Green, D.R.Morrison, hep-th/9605222
21. C.M.Hull, E.Witten, Phys. Lett. **B160** (1985) 398
22. K.Uhlenbeck, S.-T.Yau, Comm. Pure and App. Math., Vol. XXXIX (1986) 257
23. C.Vafa, Mod. Phys. Lett. **A4** (1989) 1615
24. V.V.Batyrev, alg-geom/9310003, J. Alg. Geom. **3** (1994) 493
25. M.Kreuzer, M.Nikbakht-Tehrani, in preparation
26. A.N.Schellekens, Nucl. Phys. **B366** (1991) 27
27. M.Kreuzer, C.Schweigert, Phys. Lett. **B352** (1995) 276 hep-th/9503174
28. A.N.Schellekens and S.Yankielowicz, Int. J. Mod. Phys. **A5** (1990) 2903
29. B.Gato-Rivera, A.N.Schellekens, Nucl. Phys. **B353** (1991) 519; Commun. Math. Phys. **145** (1992) 85
30. M.Kreuzer, A.N.Schellekens, Nucl. Phys. **B411** (1994) 97
31. C.Schweigert, Theor. Math. Phys. **98** (1994) 326 hep-th/9311168
32. R.Blumenhagen, R.Schimmrigk, A.Wiřkirchen, hep-th/9609167